antimony and bismuth. The results are tabulated in Table IV.

The velocity error for v_8 was calculated from Waterman's formula $\Delta v_8 = v_8\theta c_{14}/c_{44}$, after putting $c_{25}=0$, where θ is the polar misorientation angle, or $\Delta\theta$ in our notation.[10] The value for Δv_8 of antimony and bismuth was inserted in Table IV. Waterman's expression for the increase in the longitudinal velocity along the trigonal axis is $\Delta v_7 = v_7\theta^2(2c_{44}+c_{13}-c_{33})/2c_{33}(c_{33}-c_{44})$. The value for antimony is $\Delta v_7 = 0.01\times10^5$ cm/sec, and the corresponding value for bismuth 0.002×10^5 cm/sec, were added to their respective values of v_7 and inserted in Table I. Waterman's expression for Δv_7 is a systematic term and amounts to about $\frac{1}{4}\%$ of v_7 for antimony and $\frac{1}{10}\%$ for bismuth. Apparently Δv_7 cannot explain the 34% drop in χ^2 when the misorientation correction is applied to bismuth, because it is so small. The change comes about primarily for bismuth, as mentioned in the text, because of the influence of the weighting factors on the "best" values for the elastic-stiffness constants.

Equations (A5)–(A10) give the velocity errors for the 45°-cut crystal and are to first order in θ. The second-order terms in φ were neglected. There were no second-order terms in θ. The G and H terms in Eq. (A1) would add nothing to Eqs. (A5)–(A10) if they had been included. The reason is that they would occur as products and squares such as G^2, H^2, and $2GHF$, which on differentiation become $2GdG$, $2HdH$, $2HFdG$, and all equal zero because G and H contain $\cos\varphi$ ($\varphi=\pm90°$ for propagation in the bisectrix) as a coefficient.

$$\Delta v_9 = \left\{(c_{11}-c_{33}) + \frac{2(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}-c_{14})(c_{11}+c_{33}-2c_{44}) - 4(c_{13}+c_{44}-c_{14})c_{14}}{2\{(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}-c_{14})^2+(c_{13}+c_{44}-c_{14})^2\}^{1/2}}\right\} d\theta/4\rho v_9, \tag{A5}$$

$$\Delta v_{10} = (c_{66}-c_{44})d\theta/2\rho v_{10}, \tag{A6}$$

$$\Delta v_{11} = \left\{(c_{11}-c_{33}) - \frac{2(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}-c_{14})(c_{11}+c_{33}-2c_{44}) - 4(c_{13}+c_{44}-c_{14})c_{14}}{2\{(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}-c_{14})^2+(c_{13}+c_{44}-c_{14})^2\}^{1/2}}\right\} d\theta/4\rho v_{11}, \tag{A7}$$

$$\Delta v_{12} = \left\{(c_{33}-c_{11}) + \frac{2(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}+c_{14})(2c_{44}-c_{11}-c_{33}) + 4(c_{13}+c_{44}+c_{14})c_{14}}{2\{(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}+c_{14})^2+(c_{13}+c_{44}+c_{14})^2\}^{1/2}}\right\} d\theta/4\rho v_{12}, \tag{A8}$$

$$\Delta v_{13} = (c_{44}-c_{66})d\theta/2\rho v_{13}, \tag{A9}$$

$$\Delta v_{14} = \left\{(c_{33}-c_{11}) - \frac{2(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}+c_{14})(2c_{44}-c_{11}-c_{33}) + 4(c_{13}+c_{44}+c_{14})c_{14}}{2\{(\frac{1}{2}c_{11}-\frac{1}{2}c_{33}+c_{14})^2+(c_{13}+c_{44}+c_{14})^2\}^{1/2}}\right\} d\theta/4\rho v_{14}. \tag{A10}$$

The equations for the velocity error for the x and y axes contained second-order corrections, were small, and therefore were neglected.

The misorientation correction, shown in Table IV, was incorporated into the formula for Δv_i in the text, and improved the χ^2 value for bismuth from 1.409 to 0.929 while the corresponding values for antimony were worsened slightly from 1.582 to 1.708. The misorientation correction improved the equality of the trace relations for antimony and bismuth, and reduced the errors slightly for the elastic-stiffness constants of the latter, while for antimony, half were reduced and half increased slightly. The correction has at the maximum a 1% effect on the antimony elastic constants. For example, if they were not incorporated into the least-squares calculation, c_{13} would decrease by 0.2×10^{10} dyn/cm^2. Likewise for bismuth, c_{13} and c_{33} would decrease by 0.1×10^{10} dyn/cm^2.

[10] P. C. Waterman, Phys. Rev. **113**, 1247 (1959).